UNIT 8

Geometry

Just for Fun

Get My Angle

Unscramble the letters in each row to form a geometry word from this unit.

LAPARLEL ____________

TARNSUAQD ____________

NMEGSTE ____________

SETROCIB ____________

GLENA ____________

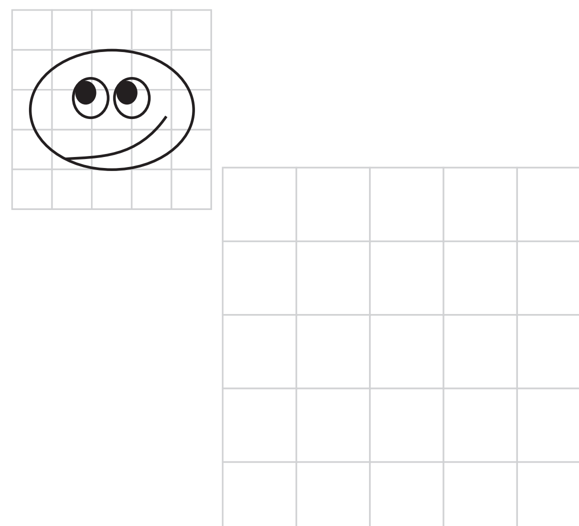
SETCIB ____________

IDLUCRANEPREP ____________

GIINRO ____________

Draw It Bigger

Enlarge the cartoon.
Draw the cartoon in the larger grid.



Sum or Product?

A Game for 2 to 4

You need two number cubes labelled 1 to 6, pencils, and paper.

Players take turns. On each turn, roll both number cubes.

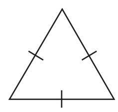
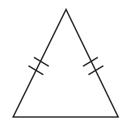
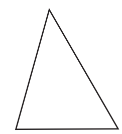
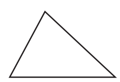
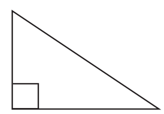
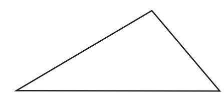
If the sum of the two numbers is greater than the product of the two numbers, the player scores 3 points.
If the product of the two numbers is greater than the sum of the two numbers, the player scores 1 point.
If the sum and product are the same, the player scores no points.

Each player has 10 turns.
The player with the most points after 10 turns is the winner.

Activating Prior Knowledge

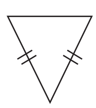
Classifying Triangles

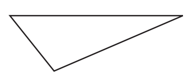
You can classify triangles by their sides, or by their angles.

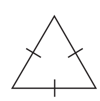
Classify by sides	 all sides equal **Equilateral**	 2 sides equal **Isosceles**	 no sides equal **Scalene**
Classify by angles	 all angles less than 90° **Acute**	 one angle of 90° **Right**	 one angle greater than 90° **Obtuse**

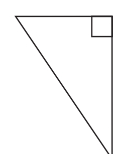
Check

1. Match each triangle to its description.









equilateral and acute isosceles and acute scalene and right scalene and obtuse

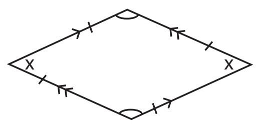
2. Draw each triangle if you can. If you cannot, explain why.

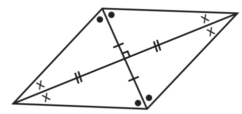
a) scalene and obtuse **b)** obtuse and acute **c)** equilateral and acute

Properties of a Rhombus

A rhombus is a parallelogram with all sides equal.
A rhombus has:

- Opposite sides parallel
- Opposite angles equal
- Diagonals that intersect at right angles
- Diagonals that bisect each other
- Diagonals that bisect the angles

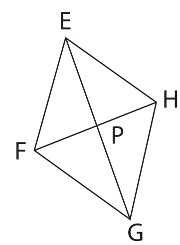




Example

For rhombus EFGH:

a) Name the opposite angles that are equal.
b) What do you know about sides EF and HG?
c) What do you know about segments EP and PH?

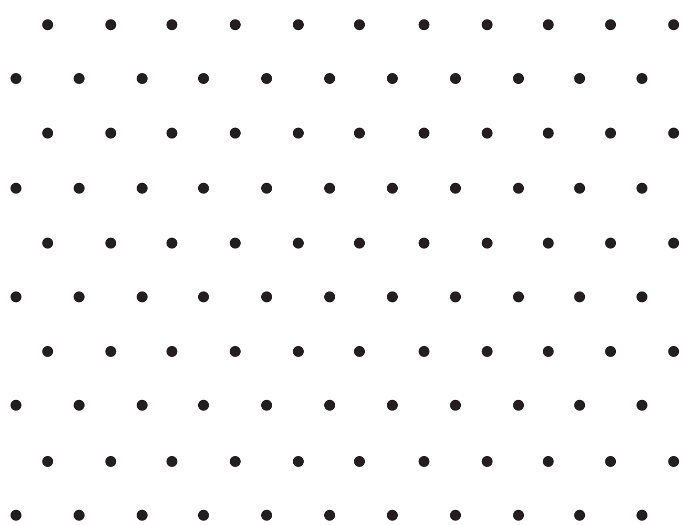


Solution

a) $\angle FEH = \angle FGH$ and $\angle EFG = \angle EHG$
b) Sides EF and HG are opposite sides, so they are equal and parallel.
c) Segments EP and PH are parts of intersecting diagonals, so these segments are perpendicular; that is, they intersect at 90^o.

Check

3. Draw a large rhombus on this dot paper.
Label it HIJK.
Draw the diagonals of the rhombus.
They intersect at point G.



a) Name all the right angles.

____

b) What is the bisector of HJ? ___

c) Which segment is twice the length of IG? ___

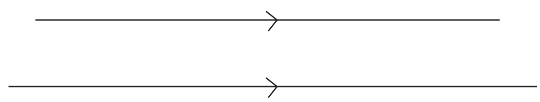
d) Which segment is one-half the length of HJ? _________

e) Which angle is equal to $\angle HIG$? ____________________

f) Which angle is equal to $\angle IHK$? ______

Quick Review

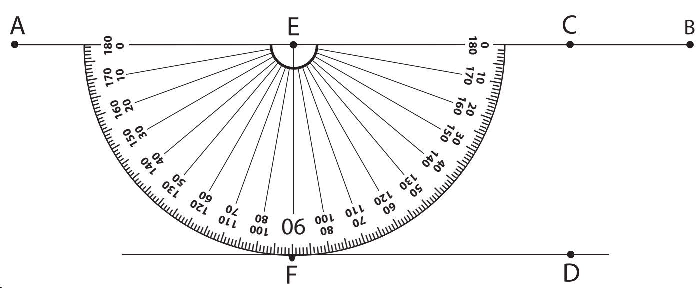
Parallel lines are lines on the same flat surface that never meet.
They are always the same distance apart.



Here are 2 strategies for drawing a line segment parallel to line segment AB.

Using a ruler and a protractor

➤ Choose a point C on line segment AB.
Place the centre of the protractor on C. Align the base line of the protractor with AB.
Mark a point D at 90°.
Repeat this step at point E to mark point F.
Draw a line through FD.
Line segment FD is parallel to AB.



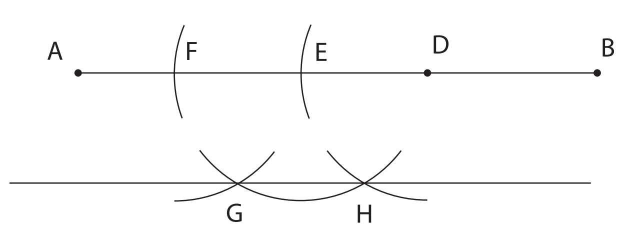
Using a ruler and a compass

➤ Mark any point D on line segment AB.
Place the compass point on D.
Draw an arc to intersect AB at E.
Place the compass point on E.
Draw an arc to intersect AB at F.

Tip

Make sure the distance between compass point and pencil point stays the same.

➤ Place the compass point on F.
Draw an arc below AB.
Place the compass point on E.
Draw an arc below AB. This arc should intersect the arc drawn from F, at G.
Place the compass point on D.
Draw an arc below AB to intersect the arc drawn from E, at H.

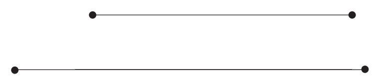


➤ Draw a line through GH.
Line segment GH is parallel to AB.

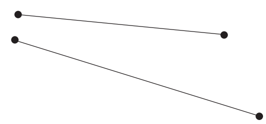
Practice

1. Are the line segments in each pair parallel?

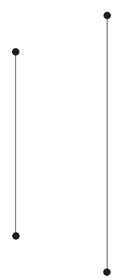
a) ______



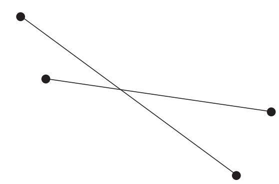
b) ______



c) ______



d) ______



2. Draw a line segment.
Use any method you like to draw a parallel line segment.
Explain your strategy. How do you know the lines are parallel?

____

____

____

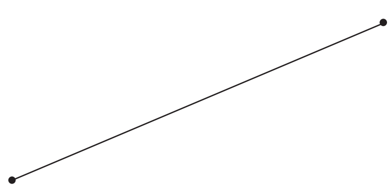
____

____

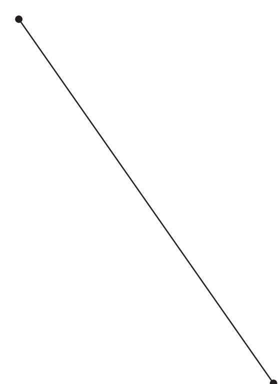
3. Look around you for examples of parallel line segments. List 6 examples.

4. Draw a line segment parallel to each segment shown.

a)



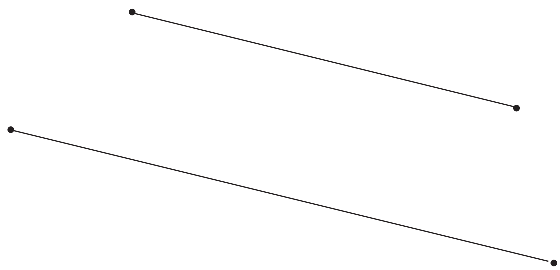
b)



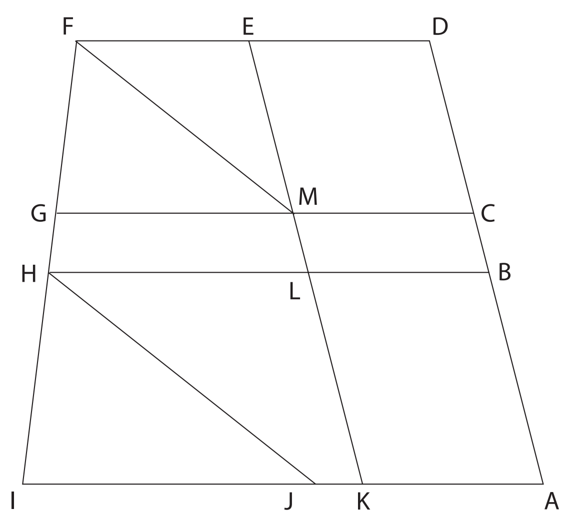
c)

d)

5. How do you know these line segments are parallel?



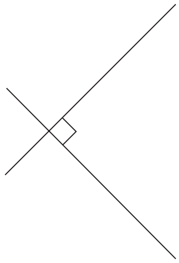
6. Look at the diagram below.
Find as many pairs of parallel line segments as you can.



Quick Review

At Home At School

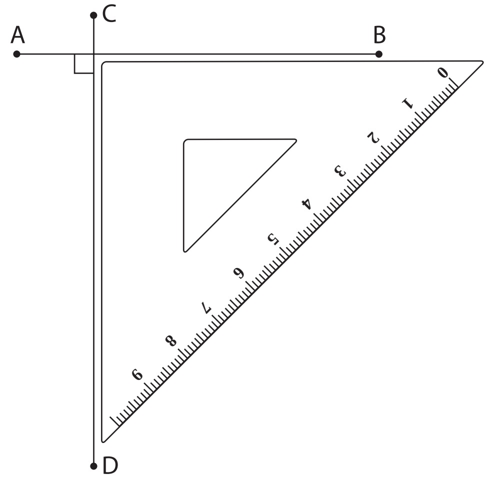
Two lines are perpendicular if they intersect at right angles.



Here are 2 strategies to draw a line segment perpendicular to line segment AB.

Using a plastic right triangle

➤ Place one of the shorter sides of the triangle along line segment AB.
Draw line segment CD along the other shorter side.
Line segment AB is perpendicular to CD.

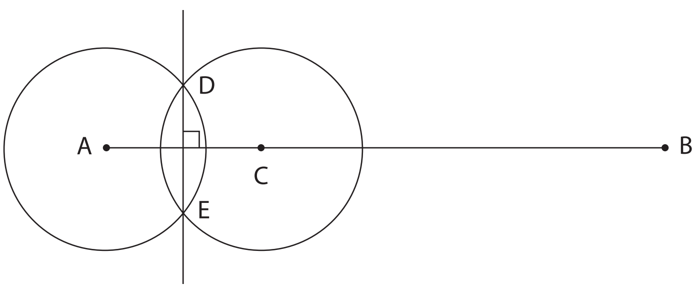


Using a ruler and a compass

➤ Mark a point C on line segment AB.
Set the compass so the distance between the compass and pencil points is greater than one-half the length of AC.
Place the compass point on A.
Draw a circle that intersects AB.

➤ Place the compass point on C.
Draw a circle to intersect the first circle you drew, at D and E.

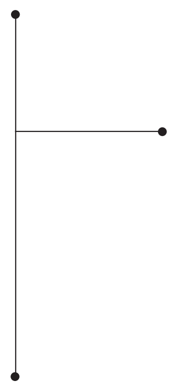
➤ Draw a line through DE.
Line segment DE is perpendicular to AB.



Practice

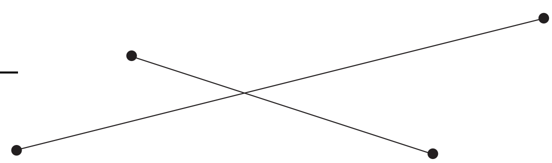
1. Are the line segments in each pair perpendicular?

a) ______

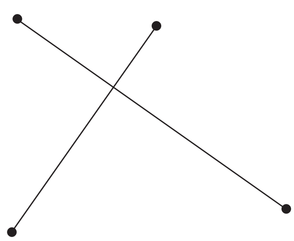


b) ______

c) ______



d) _____



2. Draw a line segment.
Use any method you like to draw a perpendicular line segment.
Explain your strategy.

____

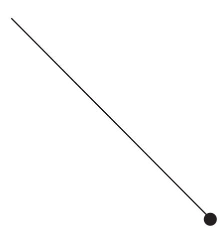
____

____

3. Look around you for examples of perpendicular line segments. List 6 examples.

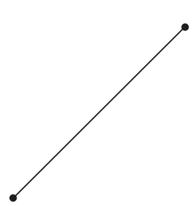
4. Draw a line segment perpendicular to each segment shown.

a)



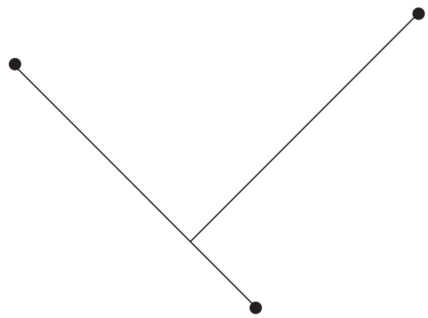
b)

c)

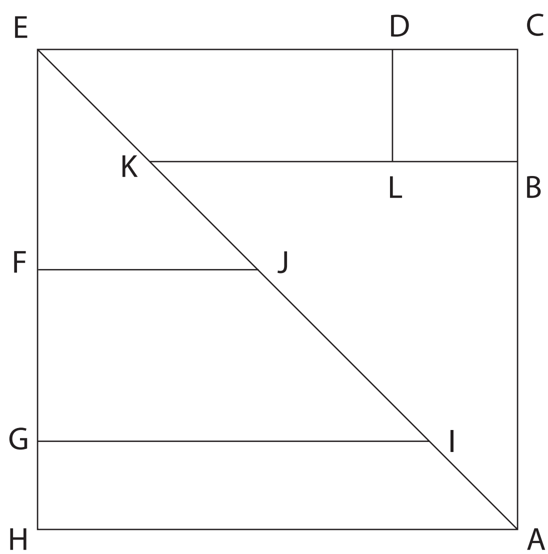


d)

5. How do you know these line segments are perpendicular?



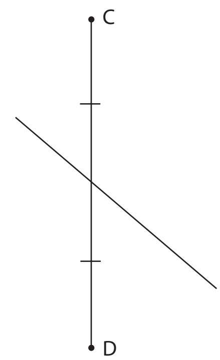
6. Look at the diagram below.
Find as many pairs of perpendicular line segments as you can.



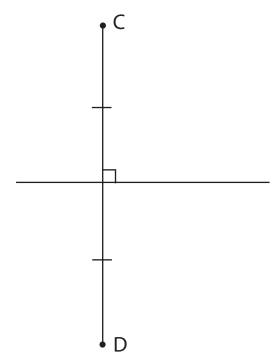
8.3 Constructing Perpendicular Bisectors

Quick Review

When you draw a line to divide a line segment into two equal parts, you *bisect* the segment. The line you drew is a *bisector* of the segment.



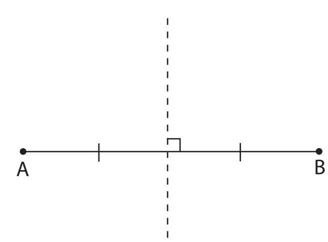
When the bisector is drawn *perpendicular* to the segment, the line is the *perpendicular bisector* of the segment.



Here are 2 strategies to draw a perpendicular bisector of line segment AB.

Using paper folding

Fold the paper so that point A lies on point B.
Crease along the fold.
Open the paper.
The fold line is the perpendicular bisector of AB.

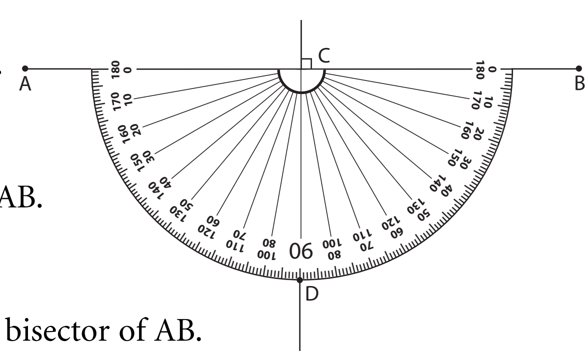


Using a ruler and a protractor

➤ Use the ruler to measure the length of AB.
Mark its midpoint C.

➤ Place the centre of the protractor on C.
Align the base line of the protractor with AB.
Mark a point D at 90°.

➤ Draw a line through CD.
The line through CD is the perpendicular bisector of AB.

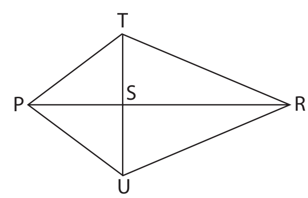


Practice

1. Match each term with its definition.

a) right angle	a line segment that joins two vertices of a shape, but is not a side
b) bisector	a 90° angle
c) perpendicular bisector	a line that divides a line segment into two equal parts
d) diagonal	a line that is perpendicular to a line segment and divides the segment into two equal parts

2. Identify the perpendicular bisector in this kite.
How do you know the segment you identified is a perpendicular bisector?



____

____

3. Draw any line segment MN.
Use any method you like to draw the perpendicular bisector of MN.
Explain your strategy.

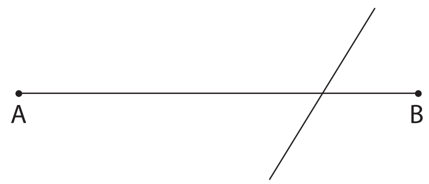
____

____

____

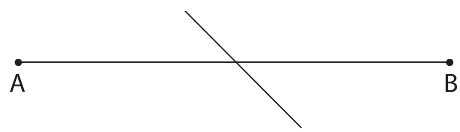
4. Has each line segment AB been bisected?
How do you know?

a) ______________________________



b) ______________________________

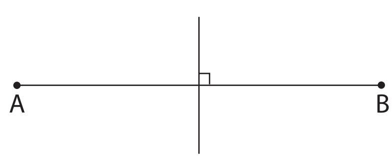
____



c) ______________________________

d) ______________________________

____



5. Look around you for examples of perpendicular bisectors. List 2 examples.

____

____

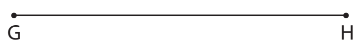
____

____

____

____

6. Draw the perpendicular bisector of line segment GH.
Draw a bisector of line segment JK.



a) Why does a line segment have many bisectors?

b) Why does a line segment have only one perpendicular bisector?

7. Draw line segment PQ 10 cm long.

a) Draw the perpendicular bisector of PQ.

b) Choose three different points A, B, and C on the bisector.

i) Measure PA and QA. ____________________

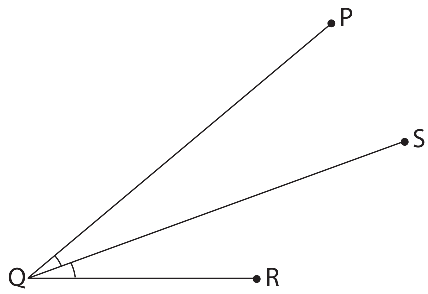
ii) Measure PB and QB. ____________________

iii) Measure PC and QC. ____________________

iv) What do you notice about the measures in parts i, ii, and iii? Explain.

Quick Review

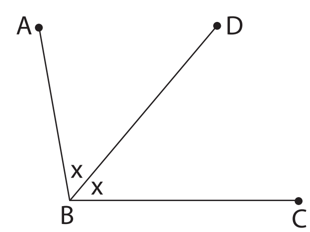
The bisector of an angle divides the angle into two equal parts.
Line segment QS is the bisector of ∠PQR because ∠PQS = ∠SQR.



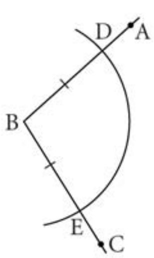
Here are 2 strategies to draw the bisector of ∠ABC.

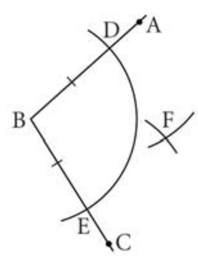
Using a Mira

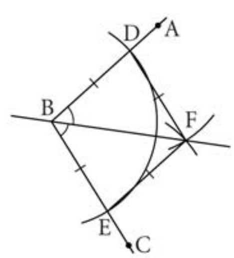
Place the Mira between the arms of the angle so that the reflection of one arm lies along the other arm.
Draw line segment BD along the edge of the Mira, through the vertex of the angle.
Line segment BD is the bisector of ∠ABC.



Using a ruler and compass

i)


ii)


iii)


i) Place the compass point on B. Draw an arc to meet BA at D and BC at E.

ii) Place the compass point on D. Draw an arc between the arms of the angle.
Keep the distance between the compass and pencil points.
Place the compass point on E. Draw an arc to meet the previous arc at F.

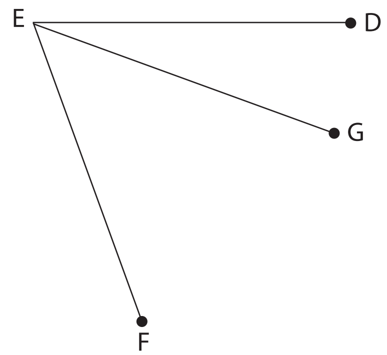
iii) Join BF.

BF is the bisector of ∠ABC.

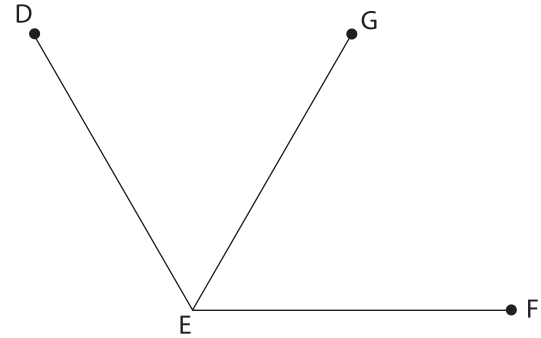
Practice

1. For each ∠DEF, is EG a bisector?

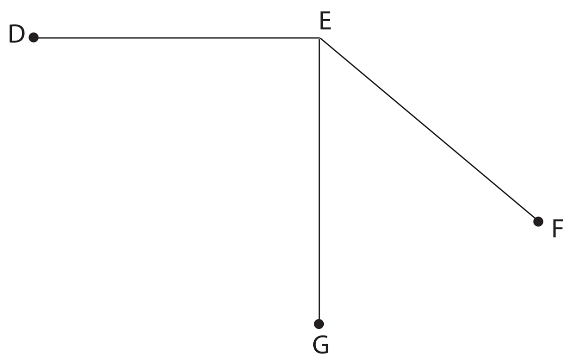
a) ___



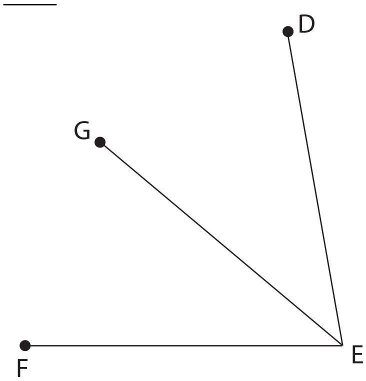
b) ___



c) ___



d) ___



2. Draw any acute ∠MNP.
Use any method you like to draw the bisector of ∠MNP.
Explain your strategy.

____

____

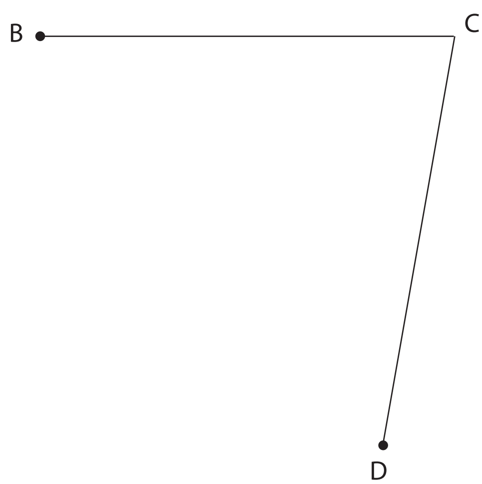
____

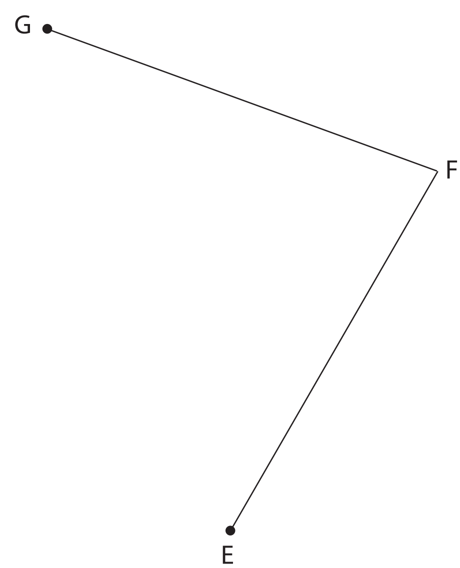
3. How can you check that the bisector you drew in question 2 is correct?

____

____

4. Use a plastic right triangle to bisect ∠BCD.
Use a ruler and compass to bisect ∠EFG.





Measure the two angles formed by each bisector.

a) Measures of angles formed by bisector of ∠BCD: ________

Measures of angles formed by bisector of ∠EFG: ________

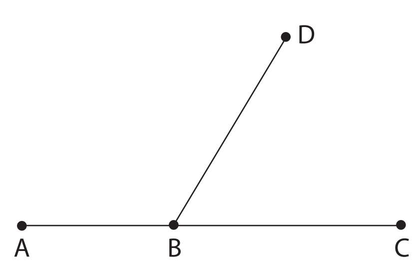
b) What do your answers from part a tell you about ∠BCD and ∠EFG?

____

5. Use a ruler and compass to draw the bisector KB of ∠ABD.
Then draw the bisector MB of ∠DBC.
Use a protractor to measure ∠KBM.

What do you notice?

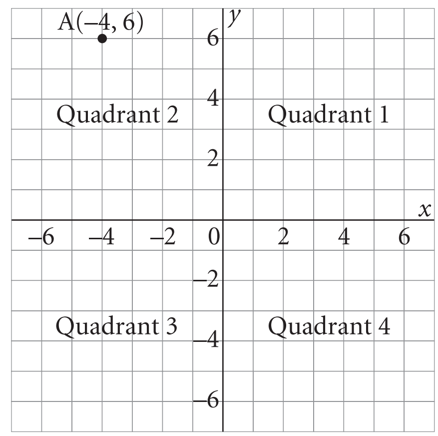
____



Graphing on a Coordinate Grid

Quick Review

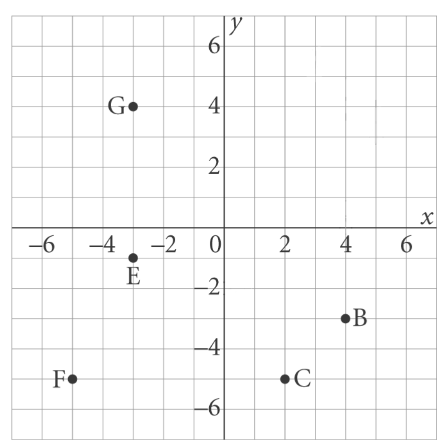
- A **coordinate grid** is formed when a horizontal number line and a vertical number line intersect at right angles at 0.



- The horizontal number line is the ***x*-axis**. The vertical number line is the ***y*-axis**. They meet at the **origin**.
- The axes divide the grid into 4 **quadrants** numbered 1, 2, 3, and 4 counterclockwise.
- Points on the axes do not belong to any quadrant.
- A point on a coordinate grid is located by an **ordered pair** of numbers. The first number, the *x*-coordinate, tells how far left or right of the origin the point is. The second number, the *y*-coordinate, tells how far up or down from the origin the point is. For example, (–4, 6) is 4 units left of the origin and 6 units up.

Practice

1. Use each letter once. Complete these descriptions for points in the diagram.



a) Point ______ is 3 units right of and 5 units up from the origin.

b) Point ______ is 5 units right of and 3 units up from the origin.

c) Point ______ has *x*-coordinate 0.

d) Point ______ has *y*-coordinate 0.

e) Points ____________________________ have the same *x*-coordinates.

f) Points ______ and ______ have equal *x*- and *y*-coordinates.

g) Points ______ and ______ are in Quadrant 4.

2. Use the diagram in question 1.
Write the coordinates of each point.

HINT

Find how far left/right of the origin and then how far up/down.

a) A (4, _______) **b)** B (_______, –3)

c) C _____________ **d)** D _____________

e) E _____________ **f)** F _____________ **g)** G _____________

h) H _____________ **i)** I _____________ **j)** J _____________

3. Plot these points on the grid.

(2, –9), (0, –5), (–4, –7), (–6, –10),

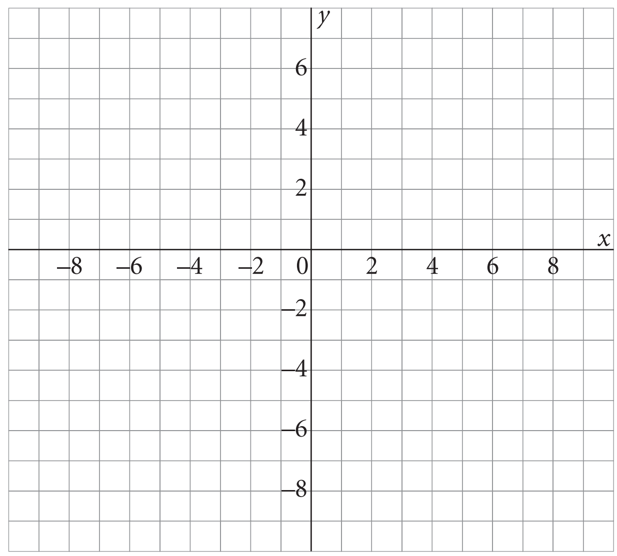
(–8, –10), (–9, –8), (–7, 0), (–5, 5),

(–6, 7), (–3, 6), (2, 3), (4, 0), (8, –4)

Join the points in the order listed.

Which animal's head did you draw?

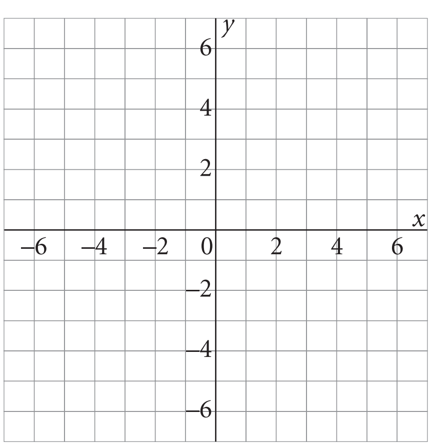
____

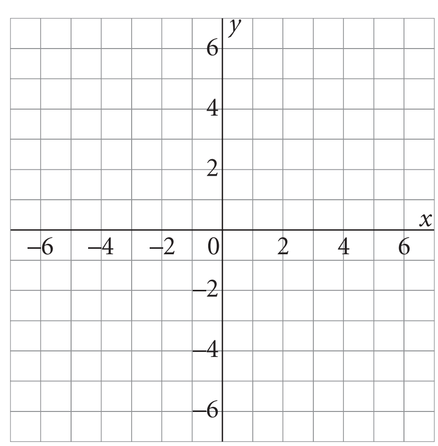


4. Graph each set of points. Join the points in order. Then join the last point to the first point.
Name the geometric shape you drew.

a) (5, 3), (5, –3), (–5, –3), (–5, 3) **b)** (–4, 0), (2, 0), (5, 3), (–1, 3)

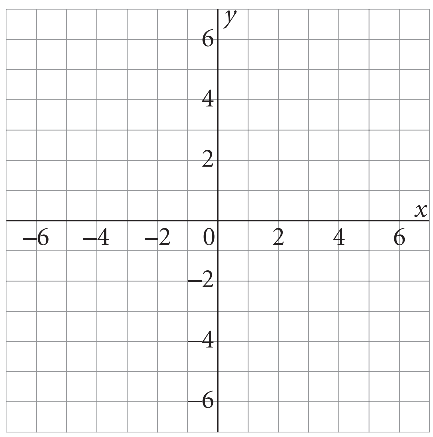
_______________________________ _______________________________





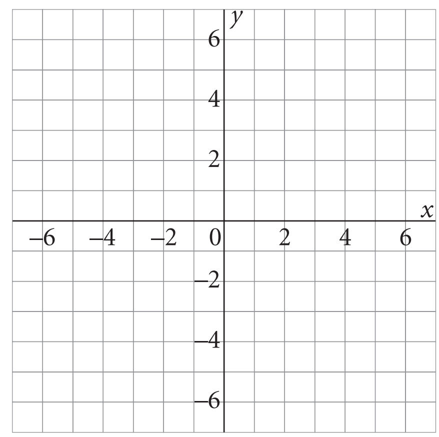
c) (–3, 4), (2, 4), (4, –2), (–4, –2)

____



d) (5, 1), (–2, –2), (–5, 1), (–2, 4)

____

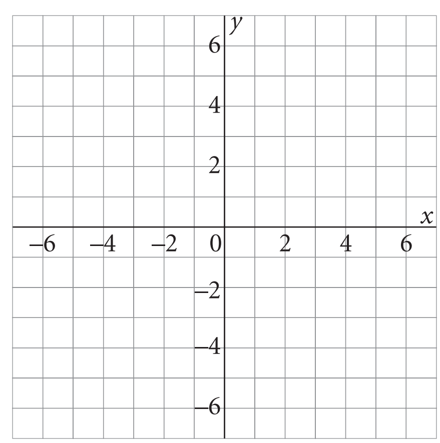


5. a) A(–4, –2), B(–4, 5), and C(3, 5) are 3 vertices of square ABCD.
Graph these points on the grid.

b) What are the coordinates of point D?

____

Graph this point on the grid.
Join the points to form square ABCD.



c) Find the area and perimeter of square ABCD.

The side length of square ABCD is ____________ units.

The area of square ABCD is ____________ square units.

The perimeter of square ABCD is ____________ units.

8.6 Graphing Translations and Reflections

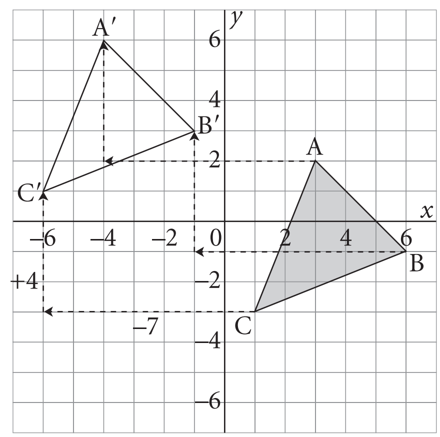
Quick Review

➤ A translation moves a shape in a straight line.
The shape and its image are congruent, and have the same orientation.

When the shape is on a square grid, the translation is described by movements right or left and up or down.

△A′B′C′ is the image of △ABC after a translation 7 units left and 4 units up.

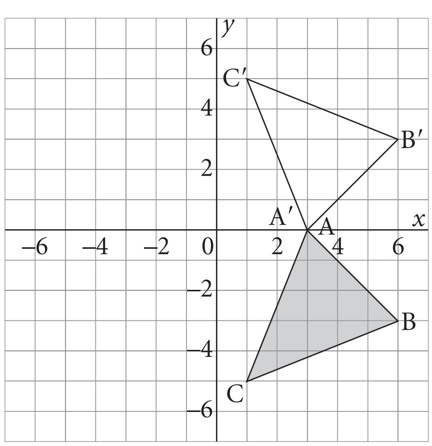
Both △ABC and its translation image △A′B′C′ are read clockwise.

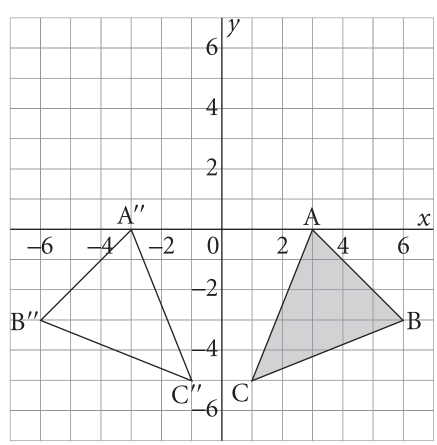


➤ A reflection creates a mirror image of a shape.
The mirror line is a line of symmetry for the shape and its image.

The shape and its image are congruent, but have different orientations.

△A′B′C′ is the image of △ABC after a reflection in the *x*-axis.
△A″B″C″ is the image of △ABC after a reflection in the *y*-axis.

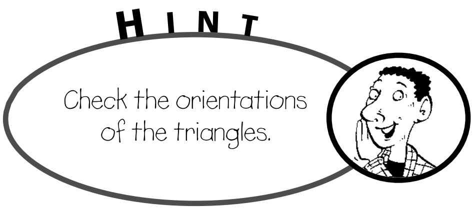


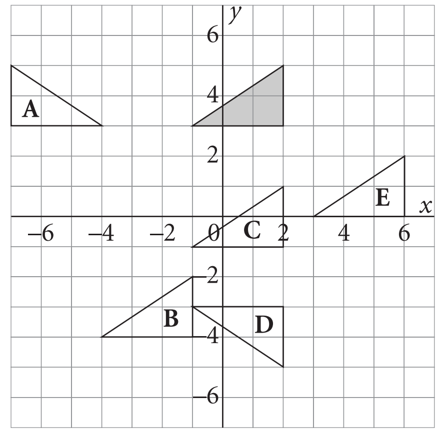


△ABC is read clockwise.
Its reflection images △A′B′C′ and △A″ B″ C″ are read counterclockwise.

Practice

1. Which triangles are translation images of the shaded triangle? Which are reflection images?





Triangles ____________ are translation images.

Triangles ____________ are reflection images.

2. **a)** Draw the image of △ABC after a translation of 5 units left and 3 units up.

b) Write the coordinates of the vertices of △ABC and its image △A′B′C′.

The image of A(1, 0) is A′(–4, 3).

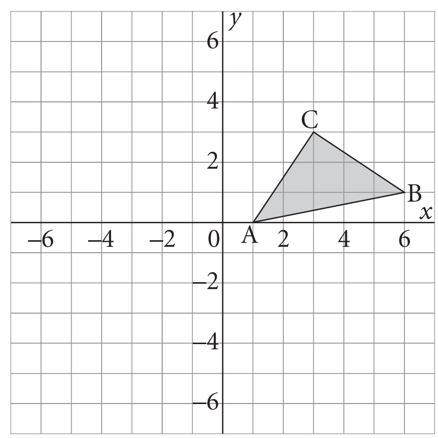
The image of B(6, 1) is B′ _________

The image of C________ is C′_________

c) For a translation 5 units left and 3 units up,

the *x*-coordinate _______________ by 5,

and the *y*-coordinate _______________ by 3.



3. Quadrilateral W′X′Y′Z′ is a translation image of quadrilateral WXYZ.

a) Describe the translation.

____

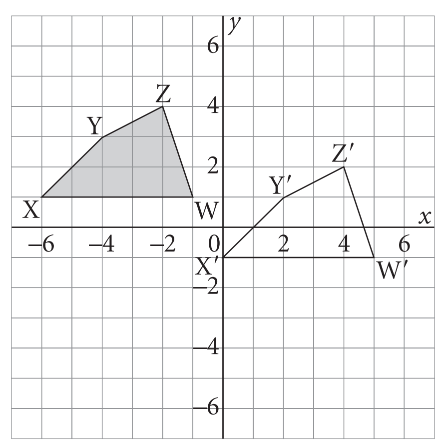
b) Write the coordinates of the vertices of the quadrilateral and its image.

The image of W_______ is W′_______

The image of X _______ is X′_______

The image of Y _______ is Y′_______

The image of Z _______ is Z′_______



4. a) Draw the image of quadrilateral KLMN:
- after a reflection in the *y*-axis. Label the image K′L′M′N′.
- after a reflection in the *x*-axis. Label the image K″L″M″N″.

> **Tip**
> *To reflect a point, find its distance from the mirror line.*

b) Write the coordinates of the vertices of KLMN and its image K′L′M′N′.

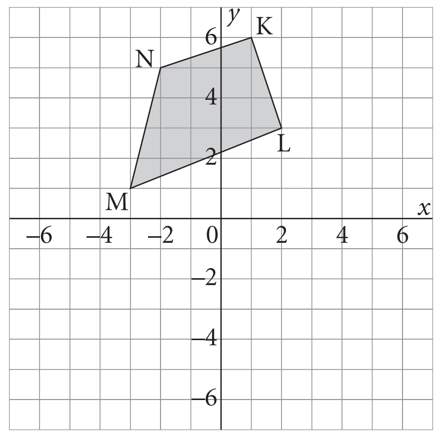


Image of K__________ is K′__________

Image of L__________ is L′__________

Image of M__________ is M′__________

Image of N__________ is N′__________

c) Write the coordinates of the vertices of KLMN and its image K″L″M″N″.

K__________ K″__________ L__________ L″__________

M__________ M″__________ N__________ N″__________

d) Complete each statement about reflection.

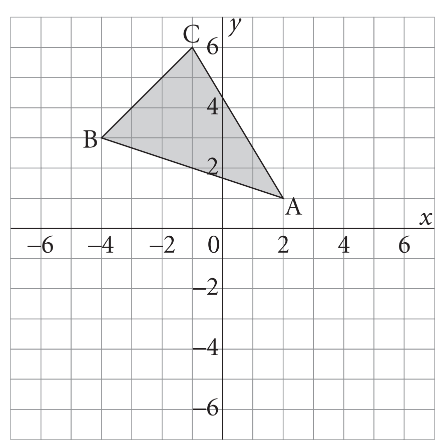
When a point is reflected in the *y*-axis, its *y*-coordinate ____________________________

and its *x*-coordinate ____________________________.

When a point is reflected in the *x*-axis, its *x*-coordinate ____________________________

and its *y*-coordinate ____________________________.

5. a) Draw the image of △ABC after a reflection in the line through P(–3, –3), O(0, 0), and R(3, 3).



b) Write the coordinates of the vertices of △ABC and its image △A′B′C′.

Image of A__________ is A′__________

Image of B__________ is B′__________

Image of C__________ is C′__________

c) What pattern do you see in the coordinates of each point and its image?

____

____

8.7 Graphing Rotations

Quick Review

➤ A rotation turns a shape about a point of rotation.

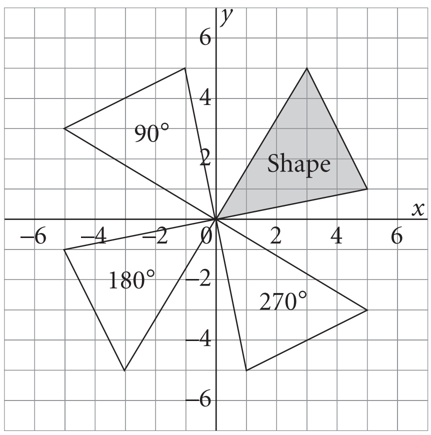
➤ Rotations can be clockwise or counterclockwise.
A counterclockwise rotation is positive.
A clockwise rotation is negative.

➤ You can use tracing paper to draw the images of a shape after a 90°, 180°, or 270° rotation about the origin on a coordinate grid.
- Trace the original shape and the axes.
- Label the positive *y*-axis on the tracing paper.
- Place a pencil point at the origin. Rotate the tracing paper counterclockwise until the positive *y*-axis coincides with the given axis.

Rotation	Positive *y*-axis coincides with . . .
90°	negative *x*-axis
180°	negative *y*-axis
270°	positive *x*-axis

- Mark the vertices of the image with a sharp pencil through the tracing paper.
- Join the vertices to draw the image of the original shape.

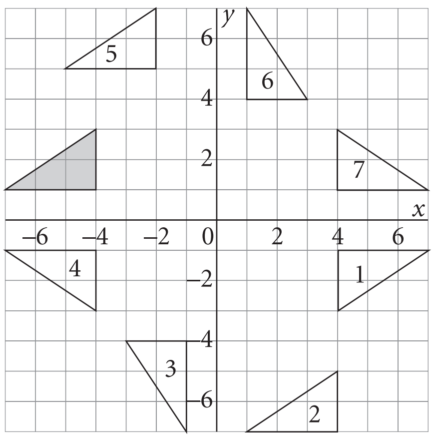
This diagram shows the images of a shape after rotations of 90°, 180°, and 270° about the origin.



➤ Each image is congruent to the original shape.

Practice

1. Match the image with each transformation of the original shaded triangle.



HINT

Reflections change the orientation of a shape, but translations and rotations do not.

Image	**Transformation of Original Triangle**
1	rotation of 90° counterclockwise about the origin
2	reflection in the *x*-axis
3	translation 2 units right and 4 units up
4	reflection in the *y*-axis
5	rotation of 180° about the origin
6	rotation of 90° clockwise about the origin
7	translation 8 units right and 8 units down

2. a) Draw the image of quadrilateral WHAT after a rotation of 90° about the origin.

b) Write the coordinates of the vertices of the original shape and its image.

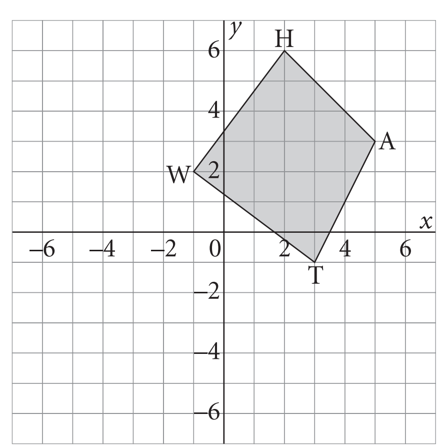
W__________ → W′__________

H __________ → H′__________

A __________ → A′__________

T __________ → T′__________

What pattern do you see in the coordinates?



____

____

3. a) Draw the image of quadrilateral WHAT after a rotation of 180° about the origin on the coordinate grid in question 2.

b) Write the coordinates of the vertices of the original shape and its image.

W__________ → W″__________

H __________ → H″__________

A __________ → A″__________

T __________ → T″__________

What pattern do you see in the coordinates?

____

____

4. a) Draw the image of quadrilateral WHAT after a rotation of –90° about the origin on the coordinate grid in question 2.

> **Tip**
> *A clockwise rotation is shown by a negative angle. A rotation of –90° is the same as a rotation of 270°.*

b) Write the coordinates of the vertices of the original shape and its image.

W__________ → W‴__________

H __________ → H‴__________

A __________ → A‴__________

T __________ → T‴__________

What pattern do you see in the coordinates?

____

____

5. Use the patterns from questions 2 to 4 to predict the coordinates of the image of K(–5, 1):

a) after a rotation of 90° about the origin. _____________

b) after a rotation of 180° about the origin. _____________

c) after a rotation of 270° about the origin. _____________

6. Write the coordinates of the vertices of the image of △ABC after a rotation of 90° about the origin.

A(3, 2) → A′__________ B(5, –4) → B′__________ C(–6, –1) → C′__________

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

parallel lines are lines on the same flat surface that never meet; the lines on this file card are parallel

perpendicular lines ______

perpendicular bisector ______

angle bisector ______

coordinate grid ______

ordered pair ______

List other mathematical words you need to know.

______

Unit Review

LESSON

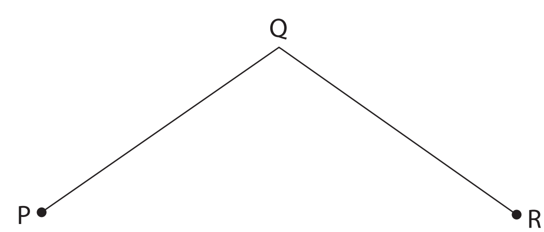
8.1 8.2 **1.** Draw line segment FG.

a) Draw a parallel line segment. Label it HJ.
Explain your strategy for drawing the parallel segment.

b) Draw the perpendicular bisector of HJ. Label it KM.

Explain your strategy for drawing the perpendicular segment.

8.3 8.4 **2.** ∠PQR is an obtuse angle.
Draw the bisector of ∠PQR.
Label it KQ.
Draw the perpendicular bisector of QR.
Label it MN.
MN intersects QR at J.

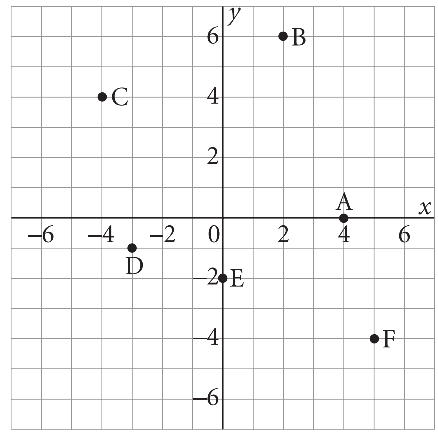


a) What do you know about ∠PQK?

b) What do you know about segment QJ?

LESSON

8.5 **3.** Use the diagram at the right.



a) The coordinates of D are ________.

b) The coordinates of F are ________.

c) Point ________ has coordinates (2, 6).

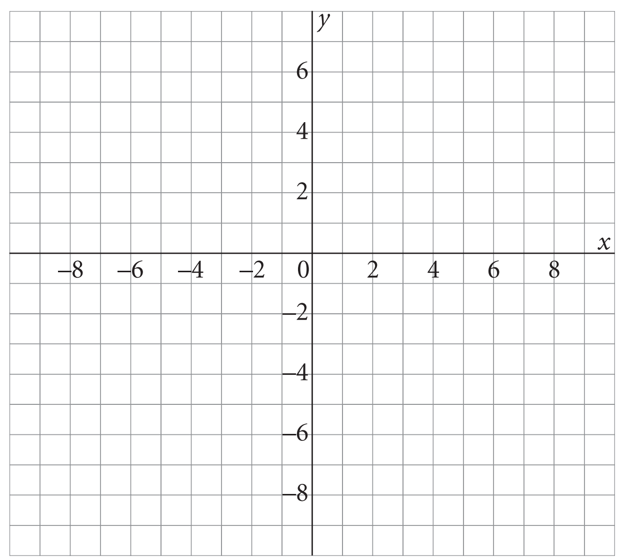
d) The coordinates of the origin are ______.

e) Point ________ has y-coordinate 0.

f) Point ________ has x-coordinate 0.

g) Point ________ is in Quadrant 2.

8.6
8.7 **4.** Plot these points on the coordinate grid: A(0, 4), B(6, 5), and C(7, –2).
Join the points to form △ABC.
On the same grid, draw the image of △ABC after each transformation.



a) A translation 9 units left and 7 units down
Label the image △A′B′C′.
Write the coordinates of the vertices of △A′B′C′.

A'________ B'________ C'________

b) A reflection in the y-axis
Label the image △A″B″C″. Write the coordinates of the vertices of △A″B″C″.

A″________ B″________ C″________

c) A rotation of –90° about the origin
Label the image △A‴B‴C‴.
Write the coordinates of the vertices of △A‴B‴C‴.

A‴________ B‴________ C‴________

Tip
A clockwise rotation is shown by a negative angle such as –90°.

How are the images alike? Different?________________________________

____

____