

Just for Fun

Secret Messages

QSFO K WKX K PSCR: POON RSW PYB K NKI.
DOKMR K WKX DY PSCR: POON RSW PYB K VSPODSWO.

Can you decipher this secret message?

Each letter in the original message has been replaced by another letter.

Each letter is replaced by the same letter every time.

No two letters are replaced by the same letter.

Each letter is shifted the same number of spaces.

Use the data below about how often each letter in the English alphabet is used.

Here is a table showing this information in a text of 1000 letters.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
73	9	30	44	130	28	16	35	74	2	3	35	25	78	74	27	3	77	63	93	27	13	16	5	19	1

Here are the frequencies of the letters in the secret message.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	2	2	3	0	1	0	0	1	0	9	0	1	3	8	7	1	5	7	0	0	1	5	2	3	0

Compare the two tables.

Shift the tallies so the greatest and least frequencies from each table match up roughly.

Each letter in the message corresponds to a letter 10 places to the left; for example, letter O in the message corresponds to E.

Decode the secret message.

Activating Prior Knowledge

Experimental Probability

$$\text{Experimental probability of an event} = \frac{\text{Number of times the event occurs}}{\text{Total number of trials}}$$

Example 1

A number cube is labelled from 1 to 6.

The cube is rolled 200 times and each outcome is recorded.

Here are the results.

Outcome	1	2	3	4	5	6
Frequency	30	40	29	28	37	36

- What is the experimental probability of rolling a 4?
- What is the experimental probability of rolling an odd number?

Solution

$$\text{Experimental probability} = \frac{\text{Number of times the event occurs}}{\text{Total number of trials}}$$

- The event, rolling a 4, occurred 28 times.

The total number of trials is 200.

So, the experimental probability of rolling a 4 is: $\frac{28}{200} = \frac{7}{50}$

- The number of times a 1, 3, or 5 occurred is: $30 + 29 + 37 = 96$

So, the experimental probability of rolling an odd number is: $\frac{96}{200} = \frac{12}{25}$

Check

Use the table in Example 1 to answer question 1.

- Find the experimental probability of each event.

a) rolling a 6: _____

b) rolling an even number: _____

c) rolling a number greater than 1: _____

Theoretical Probability

Theoretical probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

The outcomes of the event must be equally likely.

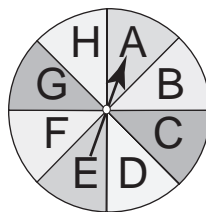
Example 2

A spinner is divided into congruent sectors.

The sectors are labelled from A to H.

The pointer is spun.

- What is the theoretical probability of the pointer landing on C?
- What is the theoretical probability of the pointer landing on a vowel?



Solution

When the pointer is spun, there are 8 possible outcomes:

A, B, C, D, E, F, G, H

The outcomes are equally likely.

- One outcome is favourable to the event of landing on C.

So, the theoretical probability of landing on C is: $\frac{1}{8}$

- There are 2 vowels: A and E

So, 2 outcomes are favourable to the event of landing on a vowel.

The theoretical probability of landing on a vowel is: $\frac{2}{8} = \frac{1}{4}$

Check

- Ten congruent marbles are placed in a bag.

There are 3 red, 4 green, 1 orange, and 2 purple.

One marble is selected at random.

What is the theoretical probability that the marble is green? _____

- A pyramid with 4 congruent triangular faces is rolled.

The faces are labelled 1, 2, 3, 4.

- What is the theoretical probability that the pyramid lands on an odd number?

- What is the theoretical probability that the pyramid lands on a number less than 4?

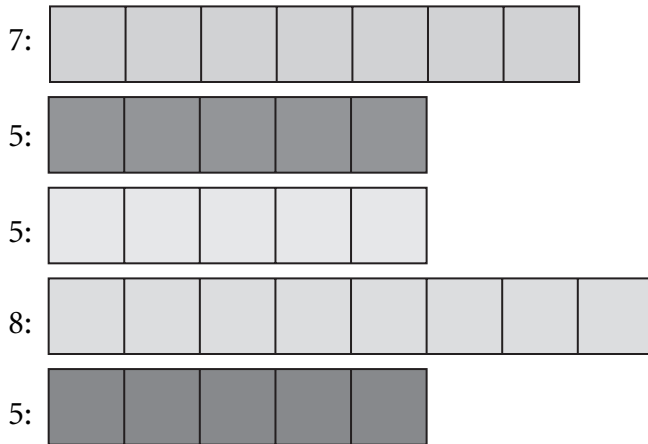


Quick Review

The **mean** is a number that can represent the centre of a set of numbers. Here are two ways to find the mean of: 7, 5, 5, 8, 5

- Use linking cubes.

Show each number with cubes:



Rearrange the cubes to make rows of equal length.
There will be 6 cubes in each row.
So, the mean of 7, 5, 5, 8, 5 is 6.

- Add, then divide.

Add: $7 + 5 + 5 + 8 + 5 = 30$

Divide by the number of numbers in the set.

There are 5 numbers in the set: $30 \div 5 = 6$

The mean of 7, 5, 5, 8, 5 is 6.

The **mode** is the number that occurs most often in a set of data.

In the set: 7, 5, 5, 8, 5; the number 5 occurs most often.

So, the mode of 7, 5, 5, 8, 5 is 5.

Both the mean and the mode are sometimes called **average**.

The mean and the mode are **measures of central tendency**.

Practice

- Use linking cubes to find the mean of each set of data.

a) 3, 6, 6, 1, 4 _____

b) 1, 3, 3, 3, 5, 3 _____

2. What is the mode of each data set in question 1?

a) _____

b) _____

3. Find the mode of each set of data.

a) 3, 7, 6, 7, 6, 4, 6 _____

b) 8, 4, 8, 2, 4, 8, 7, 8 _____

c) 1, 4, 1, 7, 1, 3, 1 _____

d) 3, 6, 3, 6, 8, 3, 3 _____

e) 19, 15, 14, 15, 15 _____

f) 94, 16, 94, 83, 83, 94 _____

4. Calculate the mean of each set of data.

a) 24, 16, 35, 52, 18 _____

b) 150, 64, 73, 125 _____

c) 20, 35, 14 _____

d) 75, 70, 36, 51, 18 _____

5. Here are the masses of 6 dogs: 25 kg, 30 kg, 25 kg, 20 kg, 25 kg, 25 kg

a) What is the mean mass? _____

b) What is the mode of the masses? _____

6. Geraldo received these marks on 5 spelling tests: 100, 98, 97, 100, 100

a) What is Geraldo's mean mark? _____

b) What is the mode of his marks? _____

7. This table shows the heights and circumferences of 5 trees.

a) What is the mean height?

b) What is the mean circumference?

Tree	Height (m)	Circumference (cm)
Oak	20	65
Elm	16	82
Maple	20	60
Birch	15	82
Poplar	9	21

c) What is the mode of the heights? _____

d) What is the mode of the circumferences? _____

8. Jocelyn has 6 birds.

Their mean age is 10.

The mode of their ages is 8.

What might their ages be?



Quick Review

The **median** of a data set is the middle number when the data are arranged in order.

The **range** of a data set tells how spread out the data are.

It is the difference between the greatest and the least numbers in the set.

- Sofia caught 7 rainbow trout.

She listed the lengths, in centimetres, from least to greatest:

37, 39, 39, 40, 43, 44, 44

The middle number is 40. The median length is 40 cm.

- Sofia caught 1 more trout. It was 42 cm long.

To find the new median, Sofia inserted the number in its correct position in the ordered list:

37, 39, 39, 40, 42, 43, 44, 44

Now there are two middle numbers: 40 and 42

The median is the mean of the two middle numbers:

$$(40 + 42) \div 2 = 41$$

The median length is now 41 cm.

- The range of the lengths is: $44 \text{ cm} - 37 \text{ cm} = 7 \text{ cm}$

Tip

The median is another measure of central tendency.

Practice

1. Arrange the numbers in each set from least to greatest.

Then, find the median and the range.

- a) 12, 18, 27, 9, 42

From least to greatest: _____

Median: _____

Range: _____

- b) 87, 76, 93, 74, 67, 91, 79

From least to greatest: _____

Median: _____

Range: _____

- c) 55, 45, 62, 71, 74, 58, 66, 58, 47

From least to greatest: _____

Median: _____

Range: _____

d) 17, 12, 18, 14, 16, 11
 From least to greatest: _____
 Median: _____ Range: _____

e) 44, 62, 17, 38, 59, 53, 48, 38
 From least to greatest: _____
 Median: _____ Range: _____

2. Find the median and the range of these amounts: \$10, \$14, \$9, \$11, \$7, \$12

Median: _____ Range: _____

3. a) Measure the arm spans and the strides of 5 people, to the nearest centimetre.
 Record your data in the table.

Name	Arm Span (cm)	Stride (cm)

b) What is the median arm span? _____

c) What is the median stride? _____

d) What is the range of the arm spans? _____

e) What is the range of the strides? _____

f) Measure one more person.
 What is the new median arm span? _____

What is the new median stride? _____

g) Has the range of arm spans changed? If so, what is the new range? _____

Has the range of strides changed? If so, what is the new range? _____

4. This list shows the numbers of books 12 students read over the summer:

8, 4, 13, 2, 4, 3, 5, 17, 7, 12, 4, 5

Find each measure:

a) mean: _____

b) median: _____

c) mode: _____

d) range: _____



Quick Review

A number in a data set that is very different from the other numbers is an outlier.

In this set of data: 35, 37, 39, 42, 82,
the outlier is 82 because it is much greater than the other numbers in the set.

The mean and median may be affected by removing the outliers.

For example, for the data set above: 35, 37, 39, 42, 82

To find the mean, add then divide:

$$(35 + 37 + 39 + 42 + 82) \div 5 = 235 \div 5 \\ = 47$$

The numbers are arranged in order, so the median is the third number: 39

With the outlier:

The mean is 47.

The median is 39.

Remove the outlier.

The data set is then: 35, 37, 39, 42

$$\text{The new mean is: } (235 - 82) \div 4 = 153 \div 4 \\ = 38.25$$

The new median is the mean of 37 and 39: $(37 + 39) \div 2 = 38$

Without the outlier:

The mean is 38.25.

The median is 38.

An outlier may result from an error in measurement or in recording.

In these cases, the outlier should be ignored when calculating averages.

Sometimes it is important to include outliers when calculating averages.

Practice

1. Identify any outliers in each data set.

a) 10, 20, 35, 35, 15, 95 _____

b) 3, 5, 8, 3, 2, 8, 5, 7, 4 _____

c) 10, 55, 61, 48, 60, 54, 97 _____

- 2.** For each data set in question 1:
- Order the data from least to greatest.
 - Calculate the mean.
 - Calculate the median.
 - Remove the outliers if they exist, then calculate the mean and median again.

a) From least to greatest: _____

Mean with outlier: _____

Median with outlier: _____

Mean without outlier: _____

Median without outlier: _____

b) From least to greatest: _____

Mean with outlier: _____

Median with outlier: _____

Mean without outlier: _____

Median without outlier: _____

c) From least to greatest: _____

Mean with outlier: _____

Median with outlier: _____

Mean without outlier: _____

Median without outlier: _____

- 3.** During one week in February, the daily snowfalls in Kingston were:
5 cm, 4 cm, 21 cm, 6 cm, 3 cm, 7 cm, 3 cm
Calculate the mean, median, and mode of the data.

Mean: _____

Median: _____

Mode: _____

4. Use the data in question 3.

Calculate the mean, median, and mode without the outlier.

Data without the outlier: _____

Mean: _____

Median: _____

Mode: _____

5. Samia has these scores on her math quizzes:

55, 89, 78, 99, 85, 83, 82, 87, 80, 78

For the mid-term report, Samia can choose between:

- using the highest average of all 10 quiz scores or
- the highest average of those scores without the outliers.

What should Samia's choice be? Justify your answer.

For all 10 scores:

Mean = _____

= _____

Arrange the 10 scores in order: _____

Median = _____

= _____

Mode = _____

The outliers are: _____

The scores without the outliers are: _____

Mean = _____

= _____

Median = _____

= _____

Mode = _____



Quick Review

The mean, median, and mode are all measures of central tendency of a set of data.

However, not all of the measures describe the data in the same way.

- Zoe's hamster has had several litters of babies.

Zoe recorded the number of babies in each litter: 17, 16, 15, 12, 5, 5, 4

There are 7 numbers in the set.

Mean: $\frac{17 + 16 + 15 + 12 + 5 + 5 + 4}{7} = \frac{74}{7}$, which is about 10.6

The mean is 10.6 babies in a litter.

The numbers of babies in order from least to greatest are: 4, 5, 5, 12, 15, 16, 17

The middle number is 12.

So, the median is 12 babies in a litter.

The number 5 occurs most often.

So, the mode is 5 babies in a litter.

Four of the 7 litters have numbers greater than 5.

So, the mode is not representative of the data.

The mean, 10.6, is not one of the data.

The median, 12, is one of the data.

The number of data greater than the median is equal to the number of data less than the median.

So, the median would be the best measure of central tendency used to describe the "average" litter size of the hamster.

- Since the mean, median, and mode are all averages, it is important to know which one best represents the data.
 - When the data represent measures such as clothing sizes, the mode best represents the data. A store needs to re-stock the sizes that sell the most.
 - When the numbers in the data set are not very different from each other, the mean is the best average.
 - When the numbers in a data set are very different, the median is the best average.

The numbers in the data set of Zoe's hamster's litter size are very different, so the median is the best average.

Practice

- The term “average” can refer to the mean, median, or mode.
Which average is likely being referred to in each case?
 - the average size of running shoes sold in one month _____
 - the average daily rainfall in Vancouver during October _____
 - the average amount spent in dollars by customers at a grocery store _____
 - the average age of people buying skateboards _____

- There are five numbers in a set of data.
The two modes are 0 and 2.
Both the median and the mean are 2.
Find the 5 numbers.

H I N T

For 5 numbers in a set of data, the median is the middle number.



- A cereal manufacturer says that each box of cereal contains an average mass of 50 g of raisins. A random check is made on 20 boxes.
The table shows the results.

Raisins per 400 g Box	
Amount (g)	Number of Boxes
48	1
49	4
50	4
51	6
52	5

- Calculate the mean, median, and mode of the data.

Mean: _____

Median: _____

Mode: _____

- Is the manufacturer’s claim acceptable?
Justify your answer.

4. A radio station has a weekly Song War between the top two hit songs. Listeners have all week to call in their votes. Each day, the station rounds the number of calls it receives to the nearest 10 and records the number of calls.

Votes for Song A and Song B		
Day	Song A	Song B
Monday	120	200
Tuesday	100	130
Wednesday	130	90
Thursday	250	80
Friday	100	200

- a) Find the mean, median, and mode votes of Song A.

Mean: _____

Median: _____

Mode: _____

- b) Find the mean, median, and mode votes of Song B.

Mean: _____

Median: _____

Mode: _____

- c) Which song is more popular? Explain your choice.



Quick Review

When the outcomes of an experiment are equally likely, the probability of an event occurring is:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Total number of outcomes}}$$

George has 15 bottles of flavoured water in the fridge. He has 7 bottles of lemon, 3 bottles of orange, and 5 bottles of raspberry. George takes a bottle without looking.

The probability that George takes a particular flavour of water can be expressed as a fraction, ratio, or percent.

The number of possible outcomes is 15.

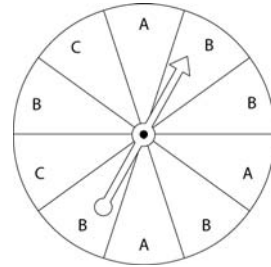
- For the probability that George takes orange:
The number of favourable outcomes is 3.
As a fraction, the probability is: $\frac{3}{15} = \frac{1}{5}$
As a ratio, the probability is: 1:5
As a percent, the probability is: $\frac{3}{15} = \frac{1}{5} = \frac{20}{100} = 20\%$
- The probability that George takes a lime-flavoured water is 0, or 0% because there is no lime-flavoured water in the fridge.
This is an **impossible event**.
- The probability that George takes a bottle that contains water is 1, or 100% because every bottle contains water.
This is a **certain event**.
- All possible probabilities lie between 0 and 1.

Tip

"Probability" is another name for "theoretical probability."

Practice

1. Suppose the pointer on this spinner is spun.



a) What is the total number of sectors on which the pointer could land? _____

b) Use a fraction, a ratio, and a percent to describe the probability of each event.

i) The pointer will land on A.

As a fraction: $\frac{\quad}{10}$

As a ratio: _____ :10

As a percent: _____

ii) The pointer will land on B.

As a fraction: _____

As a ratio: _____

As a percent: _____

iii) The pointer will land on a number.

As a fraction: _____

As a ratio: _____

As a percent: _____

iv) The pointer will land on a letter.

As a fraction: _____

As a ratio: _____

As a percent: _____

v) The pointer will *not* land on C.

As a fraction: _____

As a ratio: _____

As a percent: _____

2. Gordon has some gumballs in a bag.

He has 7 red, 5 green, 2 yellow, 4 orange, 1 black, and 6 purple gum balls.

Gordon reaches into the bag without looking and pulls out a gumball.

a) What is the total number of possible outcomes? _____

b) Write a fraction, a ratio, and a percent to describe the probability of Gordon picking each gumball listed below.

	Fraction	Ratio	Percent
purple			
black			
pink			
red or yellow			

3. Find the probability of each event.

Write each answer in any form you like.

a) Thursday immediately follows Friday. _____

b) Roll a 1, 3, or 6 on a number cube labelled 1 to 6. _____

c) Without looking, Julia picks a green ball from a bowl of balls
with 7 red, 5 yellow, and 4 green balls. _____

4. A charity sells 1000 tickets in a draw to win a new bicycle.

Find each probability.

Write each answer as a percent.

a) Lee buys 50 tickets.
The probability that Lee will win is: _____

b) Jasmine buys 20 tickets.
The probability that Jasmine will win is: _____

5. This table shows the hair colour of 30 people.

A person is picked at random.

Find each probability.

Write each answer as a fraction.

Hair Colour	Frequency
Brown	12
Blond	11
Red	5
Black	2

a) The person has black hair.

b) The person has brown hair.

c) The person does not have red hair.

d) The person has blond or brown hair.

6. Each letter in the word BRAVO is written on a separate card.

Sarah shuffles the cards and picks one without looking.

Find the probability of each outcome, as a fraction.



a) choosing "O": _____

b) choosing a vowel: _____

c) choosing a consonant: _____

d) choosing "Z": _____

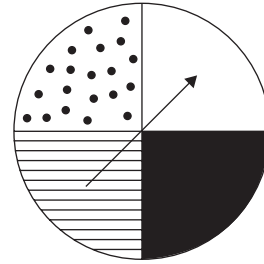


Quick Review

Two events are **independent** if the result of one event does not depend on the result of the other event. For example, tossing a coin and spinning the pointer on a spinner is an experiment with two independent events.

The outcomes of tossing a coin are: heads, tails

The outcomes of spinning the pointer on this spinner are: white, black, striped, dotted



You can use a tree diagram to show the outcomes of an experiment with 2 independent events.

List the outcomes of spinning the pointer.
For each spinner outcome, list the outcomes of tossing the coin.

There are 8 possible outcomes.

This set of outcomes is the **sample space**.

The **theoretical probability** of the pointer landing on a striped sector and the coin showing tails is: $\frac{1}{8}$

Ernesto carried out this experiment 100 times.

The event of the pointer landing on a striped sector and the coin showing tails occurred 11 times.
So, the **experimental probability** of this event is: $\frac{11}{100}$

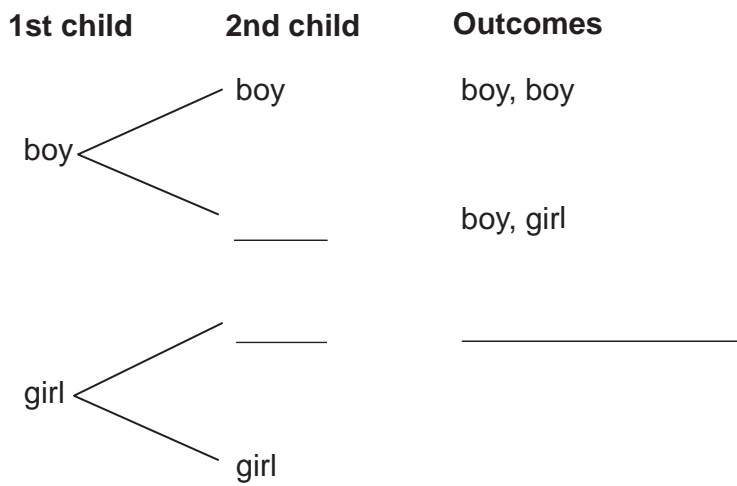
The fraction $\frac{1}{8}$ is close to the fraction $\frac{11}{100}$, so the experimental probability is close to the theoretical probability.

The greater the number of times an experiment is carried out, the closer the experimental probability may be to the theoretical probability.

Spinner	Coin	Outcomes
White	Heads	White, Heads
	Tails	White, Tails
Black	Heads	Black, Heads
	Tails	Black, Tails
Striped	Heads	Striped, Heads
	Tails	Striped, Tails
Dotted	Heads	Dotted, Heads
	Tails	Dotted, Tails

Practice

1. The theoretical probability that a new-born child is a boy is 50%, and the probability that the child is a girl is 50%.
- a) Complete the tree diagram to show the outcomes for the births of two children.



- b) List the outcomes from part a.

- c) How many outcomes are there? _____

- d) What is the theoretical probability of having a boy and a girl? _____

- e) What is the theoretical probability of having two girls? _____

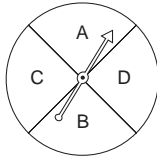
- f) In a survey of 100 families with two children, it was found that 24 families had two girls.

What is the experimental probability of having two girls? _____

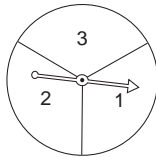
- 2.** List the sample space for each experiment.
- a) tossing a coin and rolling a tetrahedron labelled 1 to 4



- b) spinning the pointer on Spinner 1 and spinning the pointer on Spinner 2



Spinner 1



Spinner 2

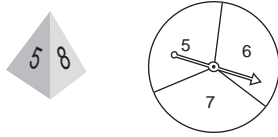
- 3.** At the cafeteria, the lunch choices of the day are:
- a cheese sandwich or a peanut butter sandwich
 - grapes or a banana or an apple
 - milk or juice
- a) Make a tree diagram to show all possible lunch orders.

- b) Suppose choices are made at random.
What is the probability that an order will include:

- i) a peanut butter sandwich? _____ ii) a banana and juice? _____
- iii) a bologna sandwich? _____ iv) a cheese sandwich, grapes, and milk _____

4. A tetrahedron is labelled 5, 6, 7, 8.

The tetrahedron is rolled and the pointer on this spinner is spun.



- a) Draw a tree diagram to show the possible outcomes.

- b) Find the theoretical probability of getting different numbers on the tetrahedron and the spinner. _____

- c) Find the theoretical probability of getting the same number on both the tetrahedron and the spinner. _____

- d) This experiment is carried out 50 times.

There were 11 outcomes where the numbers were the same.

What is the experimental probability of getting two numbers the same? ____

- e) How does the experimental probability in part d compare with the theoretical probability in part c? Explain.

In Your Words

Here are some of the important mathematical words of this unit.

Build your own glossary by recording definitions and examples here. The first one is done for you.

mean *the sum of all data in a set of numbers divided by the number of data in the set*

For example, the mean of 1, 3, 3, 5 is 3.

$1 + 3 + 3 + 5 = 12$ and $12 \div 4 = 3$.

median

outlier

theoretical probability

experimental probability

tree diagram

List other mathematical words you need to know.

Unit Review

LESSON

7.1 1. Calculate the mean and mode of each set of data.

a) The weekly allowances of ten students:

\$20, \$25, \$15, \$20, \$10, \$20, \$30, \$10, \$20, \$0

Mean: _____ Mode: _____

b) Students' scores on a spelling quiz marked out of 10:

5, 8, 8, 4, 6, 3, 10, 10, 4, 6, 7, 9, 7, 9, 9

Mean: _____ Mode: _____

7.2 2. Arrange the data in each set in order, then calculate the median and the range.

a) The heights, in centimetres, of eleven 12-year-olds:

160, 155, 162, 152, 161, 154, 153, 160, 158, 155, 159

From least to greatest: _____

Median: _____ Range: _____

b) The hours that ten grade 7 students exercised in one week:

5, 7, 18, 5, 13, 9, 4, 12, 7, 20

From least to greatest: _____

Median: _____ Range: _____

7.3 3. These data show the daily temperatures, in degrees Celsius, for two weeks in the summer in Nelson, B.C.:

23, 25, 22, 25, 28, 24, 25, 24, 25, 25, 52, 24, 20, 22

a) Find the mean, median, mode, and range for these data.

Mean: _____ Median: _____ Mode: _____ Range: _____

b) Identify the outlier. _____

Why do you think the outlier is so much greater than the other temperatures?

c) Calculate the mean, median, mode, and range without the outlier.

Mean: _____ Median: _____ Mode: _____ Range: _____

d) When reporting the average daily temperature, should the outlier be included?

Explain. _____

7.3 **4.** The times, in minutes, that 10 students spent walking home from school one day are:

7.4 20, 16, 10, 12, 22, 65, 8, 12, 18, 7

a) Calculate the mean, mode, and median times for these data.

Mean: _____ Mode: _____ Median: _____

b) Identify the outlier. _____

c) Calculate the mean, mode, and median without the outlier.

Mean: _____ Mode: _____ Median: _____

d) Which average best describes the data? Explain.

7.4 **5.** Cary scored these points in his last six basketball games: 5, 8, 10, 7, 15, 15

a) Find the mean, median, and mode scores.

Mean: _____ Median: _____ Mode: _____

b) Which measure of central tendency is Cary likely to use to persuade his coach that he is a valuable player? Explain.

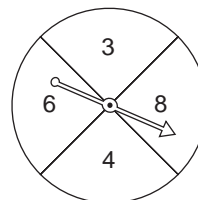
c) Which measure is the coach likely to use to help her decide if Cary is a valuable player? Explain.

LESSON

7.5 **6.** A basket of fruit contains 4 apples, 5 bananas, 6 oranges, and 10 kiwi fruits. A piece of fruit is chosen at random. Find the probability of each event. Write each probability 3 ways.

- a) An orange is picked. _____
- b) An apple or a banana is picked. _____
- c) A grape is picked. _____

7.6 **7.** Fran designs a game called *Product 24*. She makes this spinner: The pointer is spun twice. To win this game, the pointer must land on two numbers with a product of 24.



a) Complete the tree diagram to show all possible outcomes for this game.

1st Spin	2nd Spin	Outcomes
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- b) What is the total number of outcomes? _____
- c) How many favourable outcomes are there? _____
- d) What is the probability of winning Fran's game? _____